

Syncr Form Theory III: Target Inference and Certificate Construction from Data

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Abstract

This paper turns the SFT engine posture into a data-to-certificate workflow. The goal is not to “do physics automatically,” but to define the minimal inference tasks needed to produce a correct SFT artifact from observations: infer the correct target (full torus versus coupled target), estimate witness integers (degree or holonomy multiple) on an observed subsystem, and attach stability margins and convergence certificates when justified. The result is a practical interface: data and a declared model class in, witness stability certificate or obstruction report out.

Purpose

SFT is witness-first: universality claims must be expressed as objects. In practice, the obstruction to using SFT is rarely theorems; it is that real systems are observed through samples. This note isolates the inference tasks that bridge samples to certificates without changing the scope discipline of SFT.

1 Data model and declared context

A declared context specifies:

- a slice or subsystem to be observed (global slice, embedded loop, or coupled target),
- a phase observable valued in a circle or torus, with a chosen period vector $\mathbf{P}$,
- a coupling hypothesis class (none, affine diagonal, or integer-matrix constraints),
- a perturbation model and desired margins (LCM/HMR),
- whether any statistical claims are intended (RNF or NDB+MXC).

Data consist of samples of phase values, either along a loop parameter (for winding) or over time (for drive increments and coupling inference).

2 Target inference: full torus versus coupled target

3 CST inference theorem (small-integer constraints with stability)

The practical bottleneck for multi-cycle systems is identifying whether the correct target is the full torus or a coupled subtorus coset. SFT treats this as an inference problem with an obstruction fallback.

Theorem 1 (CST inference with stability or obstruction). *Fix a sampling model and a tolerance corresponding to the declared perturbation scale. There is a conservative procedure that, given sampled phase vectors in $\mathbb{T}_{\mathbf{P}}$, outputs exactly one of:*

- **CST output.** *A candidate small-integer constraint description $Ay = \Psi$ together with an error bound, yielding a coupled target CST that is stable under perturbations within the declared tolerance.*
- **Obstruction output.** *A structured reason that target inference is ill-posed at the declared tolerance (insufficient excitation, resonance/locking, ambiguity, or lack of resolution), which corresponds to a WOB class in the finite obstruction catalog.*

This theorem does not claim that all couplings are identifiable from arbitrary data. It claims that within a declared inference regime, SFT produces either a target-correct coupled description with a quantitative stability margin, or an obstruction indicating why such a description cannot be responsibly inferred.

Definition 1 (Target inference problem). *Given sampled phase vectors $y(t) \in \mathbb{T}_{\mathbf{P}}$, determine whether the image behaves as:*

- *full-rank on $\mathbb{T}_{\mathbf{P}}$, or*
- *restricted to a coupled subtorus coset CST described by integer constraints $Ay = \Psi$.*

In practice, one begins with low-complexity constraint families:

- **Affine diagonal coupling (two-cycle).** $b \ominus (\alpha \odot a) = \Psi$ for a small rational α .
- **Integer-matrix coupling (multi-cycle).** $Ay = \Psi$ for small integer matrices A .

The output of target inference is a *target declaration*: either full torus, or an explicit CST model (with baseline set to coupled Haar on CST).

4 Witness integer inference from samples

Loop samples (degree)

If a loop is sampled by phases $p_0, p_1, \dots, p_N \in [0, P)$, lift-unwrapping provides a lifted sequence $\tilde{p}_i$ with small jumps, and the degree estimate is

$$k \approx \frac{\tilde{p}_N - \tilde{p}_0}{P} \in \mathbb{Z}.$$

This is stable under the LCM margin: if two lifts remain within π , the integer cannot change.

Holonomy samples

If the observable is realized via a local lift atlas or a connection, holonomy inference uses integrated loop data to estimate the multiple m such that the loop integral equals mP . Stability is controlled by the HMR margin.

5 Certificate construction

A data-driven WSC records:

- the target (full torus or CST) and baseline rule,
- the witness integer k (degree) or m (holonomy multiple),
- stability margins (LCM/HMR),
- an optional Syncre Stability Modulus $\mathfrak{S}$ bundling these ingredients,
- pointers to the data used and the inference method used to compute the fields.

A data-driven WOB is the correct output when inference detects a finite obstruction: no sub-system, trivial winding, coupling mismatch, resonant locking, equivariance defect, and so on.

6 Convergence certificates from data

SFT treats statistical claims as artifacts:

- **RNF certificate.** Provide evidence that increments are nonresonant (up to declared tolerance) and state Haar convergence on the correct target.
- **NDB+MXC certificate.** Declare a noise-to-diffusion bridge and record a verified mixing inequality with a rate from the thresholds ledger.

7 What this enables

This paper completes a practical loop for SFT:

data + declared model class $\longrightarrow$ target inference $\longrightarrow$ witness integer inference $\longrightarrow$
WSC/WOB artifact

while preserving the central discipline: target correctness under coupling and certificate-or-obstruction outputs.