

Syncre Form Theory II: The Syncre Stability Modulus and the Classification of Cyclic Phase-Field Systems

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Abstract

This paper extends Syncre Form Theory (SFT) by introducing a single compact modulus that records the operational ingredients needed to certify forced coverage, stability under perturbations, correct targets under coupling, and (when justified) convergence rates. The modulus supports a classification program that organizes cyclic systems by target type and obstruction type, turning diverse physical realizations into a single decision architecture: either a stable witness certificate exists on the correct target, or a finite obstruction must be reported with evidence.

1 The five primitives and the engine posture

SFT is built from: sliced arenas, phase fields, frame/gauge equivalence, witness outputs (certificate or obstruction), and target correctness under coupling. The guiding posture is an engine theorem: given a declared context, the correct output is either a witness stability certificate or an obstruction report.

2 The Syncre Stability Modulus

Definition 1 (Syncre Stability Modulus). *On a declared target (full torus or coupled target), define*

$$\mathfrak{S} = (k, \text{LCM}, \text{HMR}, b, r, \lambda),$$

where k is the witness integer, LCM and HMR are stability margins, b is a distortion bound used for anti-collapse, r is effective target rank, and λ is a certified convergence rate when a diffusion/mixing model is adopted.

3 Three theorem statements phrased in $\mathfrak{S}$

Theorem 1 (Persistence threshold). *If $k \neq 0$ and $\text{LCM} < \pi$ (and/or $\text{HMR} < P/2$ in the holonomy setting), then the witness tier persists under the declared perturbation model. In engine terms, a WSC exists with the same witness integer and target.*

Theorem 2 (Anti-collapse threshold). *Assume the witness loop admits a monotone-branch decomposition and a branch bound b . Then the coverage profile admits an explicit lower bound proportional to $(|k|/b)$ on arc neighborhoods. In modulus terms, larger $|k|$ and smaller b force stronger quantitative coverage.*

Theorem 3 (Convergence threshold when justified). *If a noise-to-diffusion bridge is declared and a verified mixing inequality holds, then defects from the baseline decay exponentially at rate λ recorded in the thresholds ledger. In modulus terms, λ becomes a certified component of $\mathfrak{S}$.*

4 Stability transfer and monotonicity in the modulus

This section records the “native law” of the modulus: margins certify invariance, and admissible operations cannot change the witness integer unless a margin is crossed.

Theorem 4 (Modulus stability transfer). *Fix a declared perturbation model. Suppose a WSC exists with stability modulus $\mathfrak{S} = (k, \text{LCM}, \text{HMR}, b, r, \lambda)$.*

- *If perturbations remain within the stated margins (LCM/HMR), the witness integer k cannot change.*
- *Under any frame or gauge equivalence preserving slice correspondence and phase meaning (FEQ), the modulus components transform trivially: the certificate class is invariant.*
- *If the model includes a dynamics that preserves the invariants used by the certificate, the witness tier persists and the same k remains valid along evolution.*

Equivalently, the only way to change the certified witness integer is to cross a declared stability margin or violate the declared regime assumptions, in which case the correct output becomes a WOB rather than a WSC.

5 Classification program

SFT classifies systems by target type (full circle/torus versus coupled target) and obstruction type (finite WOB class) when certification fails. This yields four canonical classes: forced full coverage, forced coupled coverage, embedded-only coverage, and no forced coverage in the declared regime.